

$$3.1) \Sigma = \{\star, \diamond\}$$

$$\Gamma = \Sigma \cup \{\square\}$$

M TM which decides the language of words with odd number of $\star$ symbols

$$Q = \{\text{even}, \text{odd}, \text{accept}, \text{reject}\}$$

$$q_0 = \text{even}$$

Transitions:

$$\begin{aligned} (\text{even}, \star) &\mapsto (\text{odd}, \star, +1) \\ (\text{odd}, \star) &\mapsto (\text{even}, \star, +1) \\ (\text{even}, \diamond) &\mapsto (\text{even}, \diamond, +1) \\ (\text{odd}, \diamond) &\mapsto (\text{odd}, \diamond, +1) \\ (\text{even}, \square) &\mapsto (\text{reject}, \square, 0) \\ (\text{odd}, \square) &\mapsto (\text{accept}, \square, 0) \end{aligned}$$

Encoded:

$$\begin{aligned} \langle \text{even} \rangle &= 00 \\ \langle \text{odd} \rangle &= 01 \\ \langle \text{accept} \rangle &= 11 \\ \langle \text{reject} \rangle &= -1-1 \end{aligned}$$

$$\begin{aligned} \langle \star \rangle &= 01 \\ \langle \diamond \rangle &= 11 \\ \langle \square \rangle &= 00 \end{aligned}$$

$$\langle M \rangle = 00 \cdot 00 \begin{bmatrix} 00 \cdot 01 \parallel 01 \cdot 01 \cdot 1 \\ 01 \cdot 01 \parallel 00 \cdot 00 \cdot 1 \\ 00 \cdot 11 \parallel 00 \cdot 11 \cdot 1 \\ 01 \cdot 11 \parallel 01 \cdot 11 \cdot 1 \\ 01 \cdot 00 \parallel 11 \cdot 00 \cdot 0 \\ 00 \cdot 00 \parallel -1 \cdot 1 \cdot 00 \cdot 0 \end{bmatrix}$$

3.2) M from previous instruction
UTM on $\langle M \rangle \cdot \langle \diamond \star \star \rangle$

reject

3.3) Decidable languages closed under finite intersections, finite unions and complementation.

Semidecidable languages closed under finite intersections and finite unions.

$$L \text{ decidable} \Leftrightarrow (L \text{ semidecidable} \wedge L^c \text{ semidecidable})$$

3.4) N/A

