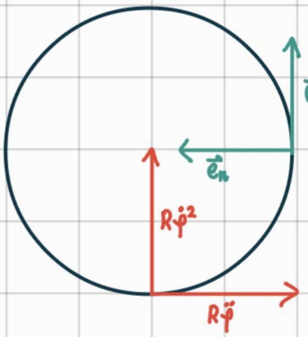
1) Izračunaj tangencialni in normalni pospešek pri gibanju po krožnici.

$$\vec{r}(t) = R(\cos(\varphi(t)), \sin(\varphi(t)))$$

$$\vec{v}(t) = \dot{\vec{r}}(t) = R(-\sin(\varphi(t))\cdot\dot{\varphi}(t), \cos(\varphi(t))\cdot\dot{\varphi}(t))$$

$$\vec{a}(t) = \dot{\vec{v}}(t) = R(-\cos(\varphi(t))\cdot\dot{\varphi}^2(t) - \sin(\varphi(t))\cdot\ddot{\varphi}(t), -\sin(\varphi(t))\cdot\dot{\varphi}^2 + \cos(\varphi(t))\cdot\ddot{\varphi}(t))$$



$\vec{e}_t = (-\sin\varphi(t), \cos\varphi(t))$ (ista smer kot hitrost)

$$\vec{a}(t) = R[(-\cos\varphi(t)\cdot\dot{\varphi}^2(t), -\sin\varphi(t)\cdot\dot{\varphi}^2(t)) + (-\sin\varphi(t)\cdot\ddot{\varphi}(t), \cos\varphi(t)\cdot\ddot{\varphi}(t))]$$

kotni pospešek

$\Rightarrow \vec{a}_t = R\ddot{\varphi}(t)\cdot(-\sin\varphi(t), \cos\varphi(t))$ $\parallel \vec{v}(t)$

$\vec{a}_n = R\cdot\dot{\varphi}^2(t)\cdot(-\cos\varphi(t), -\sin\varphi(t))$ $\parallel \vec{r}(t)$

kotna hitrost

($\varkappa = \frac{|\dot{\vec{\rho}} \times \ddot{\vec{\rho}}|}{|\dot{\vec{\rho}}|^3}$ ... torzijska ukrivljenost)

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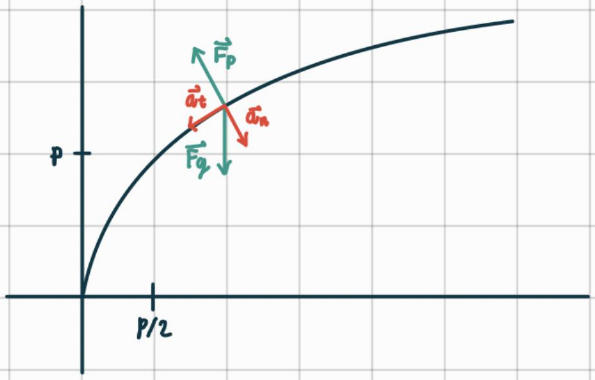
2) Gladek klanec ima obliko parabole $y^2 = 2px$. V katero točko moramo postaviti in spustiti majhno telo z maso m, da bo zapustilo parabolo v točki $(\frac{p}{2}, p)$.



$$\vec{F}_p + \vec{F}_g = \vec{a} \cdot m$$

$$\vec{F}_p + \vec{F}_g = (\vec{a}_n + \vec{a}_t) \cdot m$$

$$\vec{\varphi}(y) = \left(\frac{y^2}{2p}, y\right)$$

$$\dot{\vec{\varphi}}(y) = \left(\frac{2y}{2p}, 1\right) = \left(\frac{y}{p}, 1\right)$$

$$\ddot{\vec{\varphi}}(y) = \left(\frac{1}{p}, 0\right)$$

tangentna smer: $\vec{e}_t = -\frac{\left(\frac{y}{p}, 1\right)}{\sqrt{\frac{y^2}{p^2}+1}} = -\frac{(y, p)}{\sqrt{y^2+p^2}}$ $\qquad (\vec{e}_t \parallel \dot{\vec{\varphi}})$

normalna smer: $\vec{e}_n = \frac{(p, -y)}{\sqrt{y^2+p^2}}$ $\qquad (\vec{e}_n \perp \vec{e}_t)$

$$\dot{\vec{\rho}} \times \ddot{\vec{\rho}} = \left(\frac{y}{p}\vec{i} + \vec{j}\right) \times \left(\frac{1}{p}\vec{i}\right) = -\frac{1}{p}\vec{k}$$

ukrivljenost: $\varkappa = \frac{|\dot{\vec{\rho}} \times \ddot{\vec{\rho}}|}{|\dot{\vec{\rho}}|^3} = \frac{\frac{1}{p}}{\left(\frac{y^2}{p^2}+1\right)^{3/2}}$

2. Newtnov zakon:

tangentna smer: $\vec{F}_g \cdot \vec{e}_t = m \cdot a_t$

normalna smer: $\vec{F}_g \cdot \vec{e}_n - F_p = m \cdot a_n$

$$\vec{F}_g = (0, -mg)$$

$$\Rightarrow F_p = \vec{F}_g \cdot \vec{e}_n - m \cdot a_n =$$

$$= \frac{mgy}{\sqrt{y^2+p^2}} - m \cdot \frac{1}{p\left(\frac{y^2}{p^2}+1\right)^{3/2}} \cdot v^2 =$$

$$= \frac{m}{\left(\frac{y^2}{p^2}+1\right)^{3/2}} \cdot \left(\frac{g y \left(\frac{y^2}{p^2}+1\right)}{p} - \frac{v^2}{p} \right)$$

$$\frac{g y \left(\frac{y^2}{p^2}+1\right)}{p} - \frac{v^2}{p} = 0 \quad , \quad y = p$$

$$\frac{2yp}{p} = \frac{v^2}{p}$$

$$v^2 = 2yp$$

$$v = \sqrt{2yp}$$

$$E = T + U = \text{konst.}$$

$$mgh + 0 = mgp + m\frac{v^2}{2}$$

$$h = \frac{gp + \frac{2gp}{2}}{g} = 2p$$

$$\frac{(2p)^2}{2p} = 2p$$

Postaviti ga moramo v točko $(2p, 2p)$.

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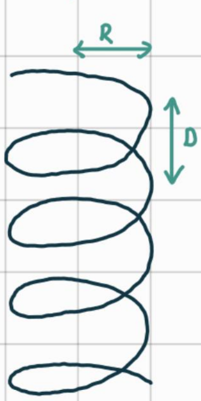
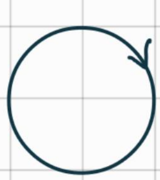
3) Na gladko vijačnico je vezana materialna točka z maso m. Določi njeno gibanje, če jo ob začetku opazovanja spustimo (brez začetne hitrosti).





$$\vec{\Phi}(\varphi) = \left(R\cos\varphi, -R\sin\varphi, -\frac{D}{2\pi}\varphi\right)$$

$$\vec{\Phi}'(\varphi) = \left(-R\sin\varphi, -R\cos\varphi, -\frac{D}{2\pi}\right)$$

$$\vec{\Phi}''(\varphi) = \left(-R\cos\varphi, R\sin\varphi, 0\right)$$

tangenta: $\Phi' = \left(-R\sin\varphi, -R\cos\varphi, -\frac{D}{2\pi}\right)$

$$= \sqrt{R^2 + \frac{D^2}{4\pi^2}} \cdot \vec{e}_t$$

binormala: $\vec{B} := \vec{\Phi}' \times \vec{\Phi}'' = \left(\frac{DR}{2\pi}\sin\varphi, \frac{DR}{2\pi}\cos\varphi, -R^2\right)$

$$= R\sqrt{\frac{D^2}{4\pi^2} + R^2} \cdot \vec{e}_{bi}$$

normala: $\vec{N} := \vec{B} \times \vec{\Phi}' = \left(R^3\cos\varphi + \frac{D^2R}{4\pi^2}\cos\varphi, -R^3\cos\varphi - \frac{D^2R}{4\pi^2}\sin\varphi, 0\right)$

$$= \left(R^3 + \frac{D^2R}{4\pi^2}\right) \cdot \vec{e}_n$$

Ukrivljenost: $\mathcal{H} := \frac{|\dot{\vec{\rho}} \times \ddot{\vec{\rho}}|}{|\dot{\vec{\rho}}|^3} = \frac{-R\sqrt{\frac{D^2}{4\pi^2} + R^2}}{\left(R^2 + \frac{D^2}{4\pi^2}\right)^{3/2}} = \frac{R}{R^2 + \frac{D^2}{4\pi^2}}$

2. Newtnov zakon:

tangentna smer: $\vec{F}_g \cdot \vec{e}_t = ma_t = m\dot{v}$

normalna smer: $\vec{F}_g \cdot \vec{e}_n + \vec{F}_p \cdot \vec{e}_n = ma_n = m\mathcal{H}v^2$

binormalna smer: $\vec{F}_g \cdot \vec{e}_{bi} + \vec{F}_p \cdot \vec{e}_{bi} = 0$

$$\vec{F}_g = (0, 0, -mg)$$

$$(t) \Rightarrow ma_t = \frac{mg\frac{D}{2\pi}}{\sqrt{R^2 + \frac{D^2}{4\pi^2}}} \Rightarrow \dot{v} = \text{konst}$$

$$\Rightarrow v(t) = \frac{gD}{2\pi\sqrt{R^2+\frac{D^2}{4\pi^2}}}\cdot t$$

$$\vec{v}(t) = \dot{\vec{r}}(t) = \left(\vec{\Phi}(\varphi(t))\right)^{\bullet} = \vec{\Phi}'(\varphi(t))\cdot\dot{\varphi}(t)$$

$$v(t) = |\vec{\Phi}'(\varphi(t))|\,\dot{\varphi}(t) = \sqrt{R^2+\frac{D^2}{4\pi^2}}\cdot\dot{\varphi}(t)$$

$$\dot{\varphi}(t) = \frac{gD}{2\pi(R^2+\frac{D^2}{4\pi^2})}\cdot t \quad\Rightarrow\quad \varphi(t) = \frac{gD}{4\pi(R^2+\frac{D^2}{4\pi^2})}\cdot t^2$$

21.11.

1) Točka se premika po ravnini tako, da ima ves čas konstantno velikost hitrosti u, smer pa oklepa kot $\alpha \in (0, 2\pi)$ z radialno smerjo.

a) Dobi trajektorijo točke.

b) Izračunaj pospešek točke.

a) $v_r = \dot{r} = u\cdot\cos\alpha$ $\qquad a_r = \ddot{r} - r\dot{\theta}^2$

$v_\theta = r\dot{\theta} = u\cdot\sin\alpha$ $\qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$r(t=0) := r_0$

$\theta(t=0) := 0$

$$r(t) = tu\cos\alpha + r_0$$

$$\dot{\theta} = \frac{u\sin\alpha}{tu\cos\alpha + r_0}$$

$$\theta\Big|_0^t = \frac{u\sin\alpha}{u\cos\alpha}\cdot\ln(tu\cos\alpha + r_0)\Big|_0^t = \tan\alpha\cdot\left(\ln(tu\cos\alpha + r_0) - \ln(r_0)\right) =$$

$$= \tan\alpha\cdot\ln\left(\frac{tu\cos\alpha}{r_0}+1\right) = \tan\alpha\cdot\ln\left(\frac{r}{r_0}\right)$$

Določimo trajektorijo v boljši obliki.

$$\ln\left(\frac{r}{r_0}\right) = \frac{\theta}{\tan\alpha}$$

$$r = r_0 e^{\frac{\theta}{\tan\alpha}}$$

b) $\dot{r} = u\cos\alpha$

$\ddot{r} = 0$

$$\dot{\theta} = \frac{u\sin\alpha}{ut\cos\alpha + r_0} = \frac{u\sin\alpha}{r}$$

$$\ddot{\theta} = -\frac{u\sin\alpha\, u\cos\alpha}{(ut\cos\alpha + r_0)^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{u^2\sin^2\alpha}{tu\cos\alpha + r_0}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{u^2\cos\alpha\sin\alpha}{tu\cos\alpha + r_0}$$