1) $\left\{ \begin{bmatrix} x & 0 & y \\ 0 & x-y & x+z \\ y & x-z & x \end{bmatrix} ; \; x, y, z \in \mathbb{R} \right\}$

Pokaži, da je to vektorski prostor in določi bazo.

$$\alpha A_{x,y,z} + \beta A_{u,v,w} =$$

$$= \alpha \begin{bmatrix} x & 0 & y \\ 0 & x-y & x+z \\ y & x-z & x \end{bmatrix} + \beta \begin{bmatrix} u & 0 & v \\ 0 & u-v & u+w \\ v & u-w & u \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha x & 0 & \alpha y \\ 0 & \alpha x - \alpha y & \alpha x + \alpha z \\ \alpha y & \alpha x - \alpha z & \alpha x \end{bmatrix} + \begin{bmatrix} \beta u & 0 & \beta v \\ 0 & \beta u - \beta v & \beta u + \beta w \\ \beta v & \beta u - \beta w & \beta u \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha x + \beta u & 0 & \alpha y + \beta v \\ 0 & \alpha x - \alpha y + \beta u - \beta v & \alpha x + \alpha z + \beta u + \beta w \\ \alpha y + \beta v & \alpha x - \alpha z + \beta u - \beta w & \alpha x + \beta u \end{bmatrix}$$

Z metodo podčrtovanja smo ugotovili, da $A \in V$.

Opazimo, da so $x, y, z \in \mathbb{R}$ neodvisne spremenljivke.

$\dim V = 3$

x	y	z	
1	0	0	*
0	1	0	*
0	0	1	*

$$A_{1,0,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A_{0,1,0} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_{0,0,1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$V = \{ x A_{1,0,0} + y A_{0,1,0} + z A_{0,0,1} \}$$
$$V = \mathrm{Lin}\{ A_{1,0,0}, A_{0,1,0}, A_{0,0,1} \}$$

$$x A_{1,0,0} + y A_{0,1,0} + z A_{0,0,1} = 0$$
$$\Downarrow$$
$$x = 0,\ y = 0 \Rightarrow z = 0$$

samo trivialna
$x = y = z = 0$

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2) $S = \{ A \in \mathbb{R}^{n \times n} ; A^T = A \}$

Dokaži, da je S vektorski podprostor in določi dimenzijo.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \in \mathbb{F}^{m \times n}$$

$$A^T = \begin{bmatrix} a_{11} & \cdots & a_{m1} \\ \vdots & & \vdots \\ a_{1n} & \cdots & a_{mn} \end{bmatrix} \in \mathbb{F}^{n \times m}$$

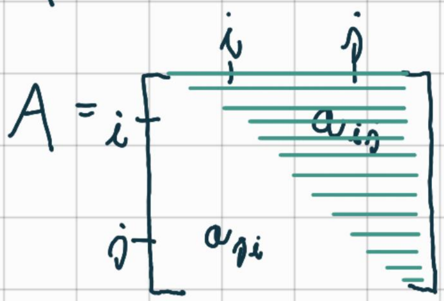
$A, B \in S$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}$$

$(X+Y)^T = X^T + Y^T$
$(\alpha X)^T = \alpha \cdot X^T$

$\Rightarrow (\alpha A + \beta B)^T = (\alpha A)^T + (\beta B)^T = \alpha A^T + \beta B^T = \alpha A + \beta B$ $\square$

$A \in S$



$A^T = A \iff \forall i,j: a_{ij} = a_{ji}$
$\hookrightarrow$ A je simetrična matrika in ima $\frac{n(n+1)}{2}$ prostih mest

F_{ij} ima 1 na ij in ji, drugod pa 0
F_{ii} ima 1 na ii, drugod pa 0

$\sum_{i \leq j} \alpha_{ij} \cdot F_{ij} = 0 \Rightarrow \forall i \leq j: \alpha_{ij} = 0$, ker je F_{ij} edina matrika, ki ima neničelni element na tem mestu

$\Rightarrow F_{ij}$ je linearno neodvisna